

MATISSE: Concept Analysis

S. Lagarde^{*a}, B. Lopez^a, R. G. Petrov^a, K.H. Hofmann^b, S.Kraus^b,
W. Jaffe^c, P. Antonelli^a, Y. Bresson^a, Ch. Leinert^d, A. Matter^a

^a UMR 6525 CNRS H. Fizeau, UNS, OCA, Nice, France,

^b Max Planck Institut für Radioastronomie, Bonn, Germany

^c Leiden Observatory, the Netherlands

^d Max Planck Institut für Astronomie, Heidelberg, Germany

ABSTRACT

MATISSE (Multi-AperTure mid-Infrared SpectroScopic Experiment) is a mid-infrared spectroscopic interferometer combining the beams of up to four UTs or ATs of the VLTI. MATISSE will be the successor to MIDI and will provide imaging capability in three spectral bands of the mid-infrared domain: L, M, and N. MATISSE will extend the astrophysical potential of the VLTI by overcoming the ambiguities that often exist in the interpretation of simple visibility measurements.

The concept of MATISSE was driven by a signal-to-noise ratio analysis aiming at comparing two basic principles that we call the global combination and the pair-wise one. We detail this comparison and explain what has led to the selected MATISSE concept: *a pair-wise $0-\pi$ multi-axial mode* [1].

Keywords: MATISSE, VLTI, Stellar Interferometry, Spectroscopy, Signal to Noise ratio, Mid-Infrared

1. INTRODUCTION

MATISSE has to comply with the following characteristics and specifications:

- 4 Telescope beam combiner
 - with the possibility to observe with 2T or 3T
- Spectral Coverage
 - Sensitivity optimized for the N and the L bands
 - other: M band
- Simultaneous observations in L(&M) and N bands
- Spectral Resolution
 - L(&M): Low=30, Medium=300-500, High=750-1500
 - N: Low=30, High=300
- Other
 - Field rotation module
 - 2D mode for acquisition
 - Polarization filters for the L band
 - Calibration devices (including phase calibration)

* S. Lagarde, Observatoire de la Côte d'Azur, BP 4229, 06304 Nice Cedex 4, France. stephane.lagarde@oca.eu

Several concepts are applicable in order to generate interferograms from 4 input beams. One of the most important constraints in the mid-infrared is the presence of a strong thermal background level. The following SNR comparison is driving the best concept selection between two different main alternatives that we call:

- The global combination,
- The pair-wise combination.

The global combination assumes that all the beams are mixed together to produce a common interference pattern. While the pair-wise combination mixes the beams pair by pair. An AMBER-like instrument is a global combination and a MIDI-like instrument is at the same time a pair-wise combination and a global one (since it has only 2 input beams).

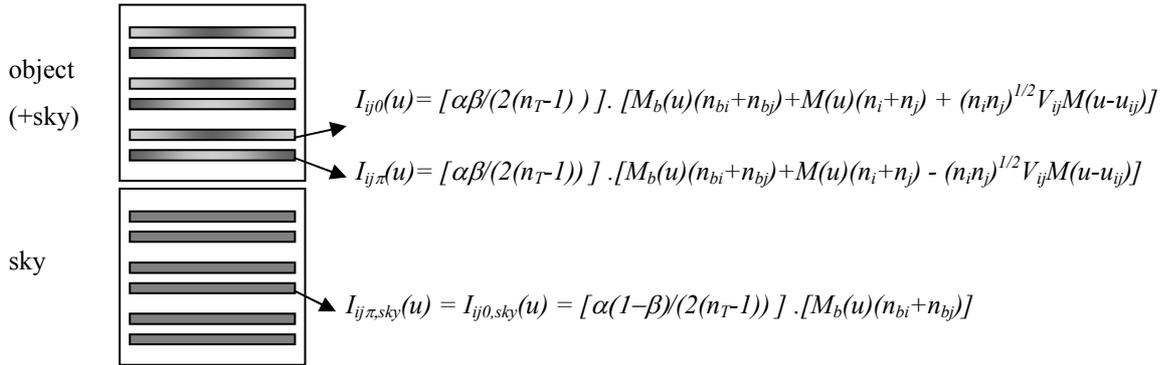
Several important related issues are to be considered. There are here listed:

- Should Multi-axial (AMBER like [2]) or co-axial (MIDI like [3]) beam superimposition preferred?
- Should MATISSE generate specific photometric images?
- Should a specific second π phase shift interferogram (existing in MIDI) be created?

2. PAIRWISE 0- π CASE

2.1 Description

The following drawing gives a sketch of the signal encoding onto the detector with the corresponding equations of the interferometric signal in the Fourier space. This in the case of observations with a pair-wise and co-axial combination with chopping (a time fraction of the observation is made on the object, the other fraction on the sky).



These equations are valid:

- In the co-axial case. This is totally equivalent to the fringes obtained in the MIDI mode.
- In the multi-axial case. It is equivalent to the co-axial case if a π phase shift is created between the two interferograms produced by each pair of beam. In this case, $u_{ij} = B_{ij}/\lambda$ where B_{ij} is the separation between the beams encoding the fringe spacing.

i and j are the index of beams. α is the proportion of the flux used in the interferometric channel, $(1-\alpha)$ for photometric channel. β corresponds to the chopping ratio. It is the fraction of time spent on source, $(1-\beta)$ on sky. n_T is the number of telescope beams used. n_{bi} and n_{bj} are the numbers of photons produced by the thermal background. n_i and n_j are the numbers of photons produced by the observed source.

$M_b(u)$ is the background function filtered by the interferometer in the Fourier space, $M(u)$ is the low frequency peak of the interferometer transfer function, $M(u-u_{ij})$ is the fringe peak of the interferometer transfer function at the spatial frequency u_{ij} . $M(0)=M_b(0)=1$. V_{ij} is the source visibility.

If a part of the flux is removed to estimate the photometry, each photometric channel when observing on source is described by,

$$P_i(u)=(1-\alpha)\beta \cdot [M_b(u)n_{bi}+M(u)n_i]$$

When observing off source (sky), the interferometric channels give:

$$I_{ij0,sky}(u) = I_{ij\pi,sky}(u) = [\alpha(1-\beta)/(2(n_T-1))] \cdot [M_b(u)(n_{bi}+n_{bj})]$$

And the photometric channels give:

$$P_{i,sky}(u)=(1-\alpha)(1-\beta) \cdot [M_b(u)n_{bi}]$$

2.2 Coherent flux estimation

The thermal background independent coherent flux, $C_{ij} = (n_i n_j)^{1/2} V_{ij}$, can be extracted from the following estimator :

$$E(C_{ij}) = [(n_T-1) / \alpha\beta] \cdot [I_{ij0}(u_{ij}) - I_{ij\pi}(u_{ij})]$$

The variance of this estimator is:

$$\sigma_{C_{ij}}^2 = [(n_T-1)^2 / \alpha^2 \beta^2] \cdot [2 \cdot \sigma_I^2]$$

With σ_I^2 (variance of each interferometric image):

$$\sigma_I^2 = [\beta\alpha/2(n_T-1)] \cdot (n_{bi}+n_{bj}+n_i+n_j) + n_{px}\sigma_D^2$$

Where σ_D is the read out detector noise per pixel.

In the multi-axial case, n_{px} represents directly the number of pixels used. In the co-axial case, the number of pixels used is less important (no fringe to sample spatially). But this number has to be multiply by n_s which is the number of steps necessary to produce each of the interferograms,

The fluxes of the two beams are approximately equivalent: $n_{bi}=n_{bj}=n_b$ and $n_i=n_j=n$

From this coherent flux estimator, the variance $\sigma_{C_{ij}}^2$ can be calculated from :

$$\sigma_{C_{ij}}^2 = [2(n_T-1)/\alpha\beta] \cdot (n_b+n) + [2(n_T-1)^2/\alpha^2\beta^2] \cdot (n_{px}\sigma_D^2) \quad (1)$$

In the case of the thermal background regime:

$$\sigma_{C_{ij}}^2 = [2(n_T-1)/\alpha\beta] \cdot n_b \quad (2)$$

2.3 Photometric flux estimation

With specific photometric channels

The photometric information can be extracted directly from the photometric channels by the following estimator:

$$E(n_i) = P_i(0)/(1-\alpha)\beta - P_{i,sky}(0)/(1-\alpha)(1-\beta)$$

The variance of this estimator is:

$$\sigma_{n_i}^2 = \sigma_{P_i}^2/(1-\alpha)^2\beta^2 + \sigma_{P_{i,sky}}^2/(1-\alpha)^2(1-\beta)^2$$

With σ_P^2 (variance of each photometric image) :

$$\begin{aligned}\sigma_{P_i}^2 &= (1-\alpha)\beta(n_{b_i}+n_i) + n_{px}\sigma_D^2 \\ \sigma_{P_i,sky}^2 &= (1-\alpha)(1-\beta)(n_{b_i}) + n_{px}\sigma_D^2 \\ &\Rightarrow\end{aligned}$$

$$\sigma_{n_i}^2 = n/(1-\alpha)\beta + n_b/(1-\alpha)\beta(1-\beta) + n_{px}\sigma_D^2/\beta$$

In the case of the thermal background regime:

$$\sigma_{n_i}^2 = n_b/(1-\alpha)\beta(1-\beta) \quad (3)$$

Without specific photometric channels

One of major advantages of the pairwise combination is the possibility to extract the photometric information without using specific photometric channels.

If 3 or more input telescope beams ($n_T \geq 3$) are used, $n_T(n_T-1)/2$ pairs or equations of the above type are produced, then the individual unknown photometries can be deduced.

In the case of 3 telescopes ($n_T=3$), without photometric channels ($\alpha=1$) and assuming a chopping on sky with $\beta=0.5$, the equations of the interferograms become:

$$\begin{aligned}I_{ij0}(u) &= (1/8) [M_b(u)(n_{b_i}+n_{b_j})+M(u)(n_i+n_j)+(n_i n_j)^{1/2} V_{ij} M(u-u_{ij})] \\ I_{ij\pi}(u) &= (1/8) [M_b(u)(n_{b_i}+n_{b_j})+M(u)(n_i+n_j) -(n_i n_j)^{1/2} V_{ij} M(u-u_{ij})] \\ I_{ij0,sky}(u) &= I_{ij\pi,sky}(u) = (1/8) [M_b(u)(n_{b_i}+n_{b_j})]\end{aligned}$$

The photometry estimator of n_i for instance becomes then:

$$E(n_i) = 2(I_{120}(0)+I_{12\pi}(0)+I_{130}(0)+I_{13\pi}(0)-I_{230}(0)-I_{23\pi}(0)) - 2(I_{120,sky}(0)+I_{12\pi,sky}(0)+I_{130,sky}(0)+I_{13\pi,sky}(0)-I_{230,sky}(0)-I_{23\pi,sky}(0)) = n_i$$

The variance $\sigma_{n_i}^2$ is :

$$\sigma_{n_i}^2 = 4(\sigma_{1120}^2 + \sigma_{112\pi}^2 + \sigma_{1130}^2 + \sigma_{113\pi}^2 + \sigma_{1230}^2 + \sigma_{123\pi}^2 + \sigma_{1120,sky}^2 + \sigma_{112\pi,sky}^2 + \sigma_{1130,sky}^2 + \sigma_{113\pi,sky}^2 + \sigma_{1230,sky}^2 + \sigma_{123\pi,sky}^2)$$

The variance of each image is given by:

$$\begin{aligned}\sigma_{I_{ij0}}^2 &= \sigma_{I_{ij\pi}}^2 = (1/8) (n_{b_i} + n_{b_j} + n_i + n_j) + (n_{px}\sigma_D^2) \\ \sigma_{I_{ij0,sky}}^2 &= \sigma_{I_{ij\pi,sky}}^2 = (1/8) (n_{b_i} + n_{b_j}) + (n_{px}\sigma_D^2)\end{aligned}$$

If the detector noise is negligible:

$$\sigma_{n_i}^2 = 4(n_{b_1}+n_{b_2}+n_{b_3}) + 2(n_1+n_2+n_3)$$

The fluxes of the 3 beams are approximately equivalent: $nb_1=nb_2=nb_3=nb$ and $n_1=n_2=n_3=n$

$$\sigma_{n_i}^2 = 12n_b + 6n$$

In the thermal background regime:

$$\sigma_{n_i}^2 = 12n_b \quad (4)$$

In the case of 4 telescopes ($n_T=4$), without photometric channels ($\alpha=1$) and assuming a chopping on sky with $\beta=0.5$, the equations of the interferograms become:

$$I_{ij0}(u) = (1/12) [M_b(u)(n_{bi}+n_{bj})+M(u)(n_i+n_j)+(n_i n_j)^{1/2} V_{ij} M(u-u_{ij})]$$

$$I_{ij\pi}(u) = (1/12) [M_b(u)(n_{bi}+n_{bj})+M(u)(n_i+n_j) -(n_i n_j)^{1/2} V_{ij} M(u-u_{ij})]$$

$$I_{ij0,sky}(u) = I_{ij\pi,sky}(u) = 1/12 [M_b(u)(n_{bi}+n_{bj})]$$

$$\sigma_{ij0}^2 = \sigma_{ij\pi}^2 = (1/12) (n_{bi} + n_{bj} + n_i + n_j) + (n_s n_{px} \sigma_D^2)$$

$$\sigma_{ij0,sky}^2 = \sigma_{ij\pi,sky}^2 = (1/12) (n_{bi} + n_{bj}) + (n_s n_{px} \sigma_D^2)$$

The photometry estimator of n_l could be:

$$E(n_l) = 2(I_{120}(0) + I_{12\pi}(0) + I_{130}(0) + I_{13\pi}(0) + I_{140}(0) + I_{14\pi}(0) - 1/2(I_{230}(0) + I_{23\pi}(0) + I_{240}(0) + I_{24\pi}(0) + I_{340}(0) + I_{34\pi}(0)) - 2(I_{120,sky}(0) + I_{12\pi,sky}(0) + I_{130,sky}(0) + I_{13\pi,sky}(0) + I_{140,sky}(0) + I_{14\pi,sky}(0)) - 1/2(I_{230,sky}(0) + I_{23\pi,sky}(0) + I_{240,sky}(0) + I_{24\pi,sky}(0) + I_{340,sky}(0) + I_{34\pi,sky}(0)))$$

The variance $\sigma_{n_l}^2$ is:

$$\sigma_{n_l}^2 = 4(\sigma_{1120}^2 + \sigma_{112\pi}^2 + \sigma_{1130}^2 + \sigma_{113\pi}^2 + \sigma_{1140}^2 + \sigma_{114\pi}^2 + \sigma_{1120,sky}^2 + \sigma_{112\pi,sky}^2 + \sigma_{1130,sky}^2 + \sigma_{113\pi,sky}^2 + \sigma_{1140,sky}^2 + \sigma_{114\pi,sky}^2) + (\sigma_{1230}^2 + \sigma_{123\pi}^2 + \sigma_{1240}^2 + \sigma_{124\pi}^2 + \sigma_{1340}^2 + \sigma_{134\pi}^2 + \sigma_{1230,sky}^2 + \sigma_{123\pi,sky}^2 + \sigma_{1240,sky}^2 + \sigma_{124\pi,sky}^2 + \sigma_{1340,sky}^2 + \sigma_{134\pi,sky}^2)$$

If the detector noise is negligible:

$$\sigma_{n_l}^2 = 6n_{b1} + 3(n_{b2} + n_{b3} + n_{b4}) + 3n_l + 3/2(n_2 + n_3 + n_4)$$

The fluxes of the 4 beams are approximately equivalent:

$$\sigma_{n_l}^2 = 15n_b + (15/2)n$$

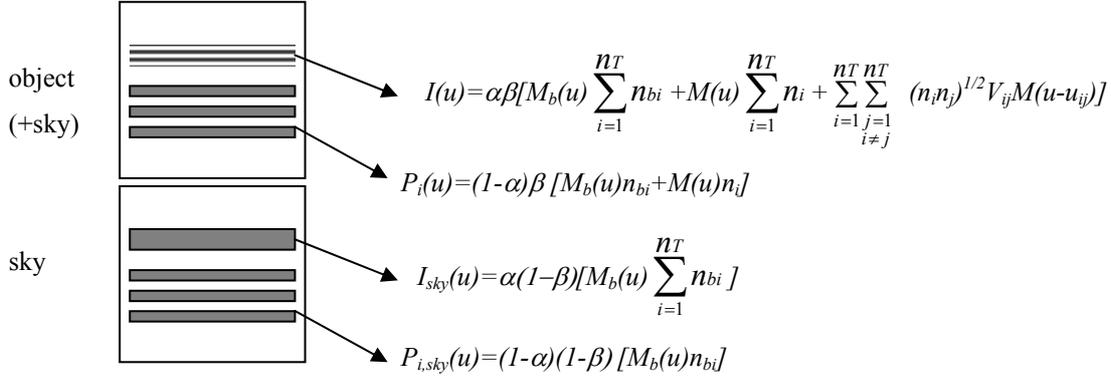
In the case of thermal background regime:

$$\sigma_{n_l}^2 = 15n_b \tag{5}$$

3. GLOBAL CASE

3.1 Description

The following drawing gives a sketch of the signal encoding onto the detector and the corresponding signal equations in the Fourier space for an observations with a global multi-axial combination with photometric images and with chopping.



In the multi-axial global case, the interferogram is defined by :

$$I(u) = \alpha\beta [M_b(u) \sum_{i=1}^{nT} n_{bi} + M(u) \sum_{i=1}^{nT} n_i + \sum_{i=1}^{nT} \sum_{\substack{j=1 \\ i \neq j}}^{nT} (n_i n_j)^{1/2} V_{ij} M(u - u_{ij})]$$

With $u_{ij} = B_{ij}/\lambda$ where B_{ij} is the separation between the beams.

Each photometric beam when observing on source is described by,

$$P_i(u) = (1 - \alpha)\beta [M_b(u) n_{bi} + M(u) n_i]$$

The interferometric channels when observing off source (sky) give,

$$I_{sky}(u) = \alpha(1 - \beta) [M_b(u) \sum_{i=1}^{nT} n_{bi}]$$

The photometric channels when observing off source (sky) give,

$$P_{i,sky}(u) = (1 - \alpha)(1 - \beta) [M_b(u) n_{bi}]$$

3.2 Coherent flux estimation

Then the coherent flux can be extracted from the following estimators considering 3 cases:

- “Clear peak” case: It corresponds to a fringe peak without contamination by the energy peak and the thermal background peak : $M_b(u_{ij}) = 0$ and $M(u_{ij}) = 0$

$$E(C_{ij}) = I(u_{ij}) / \alpha\beta$$

Leading to a variance $\sigma^2_{C_{ij}}$,

$$\sigma_{Cij}^2 = \sigma_I^2 / \alpha^2 \beta^2 \Rightarrow \sigma_{Cij}^2 = \sum_{i=1}^{nT} (n_{bi} + n_i) / \alpha \beta + n_{pi} \sigma_D^2 / \alpha^2 \beta^2$$

The fluxes of the two beams are approximately equivalent: $n_{bi}=n_{bj}=nb$ and $n_i=n_j=n$

$$\sigma_{Cij}^2 = n_T(n_b+n) / \alpha \beta + n_{px} \sigma_D^2 / \alpha^2 \beta^2$$

In the case of thermal background regime:

$$\sigma_{Cij}^2 = n_T \cdot n_b / \alpha \beta \quad (6)$$

- “Thermal background contamination peak” case: It corresponds to a fringe peak with contamination by the thermal background peak only: $M_b(u_{ij}) \neq 0$ and $M(u_{ij}) = 0$. It is the most probable case in the thermal background regime.

$$E(C_{ij}) = I(u_{ij}) / \alpha \beta - I_{sky}(u_{ij}) / \alpha (1-\beta)$$

Leading to a variance σ_{Cij}^2 ,

$$\sigma_{Cij}^2 = \sigma_I^2 / \alpha^2 \beta^2 + \sigma_{Isky}^2 / \alpha^2 (1-\beta)^2 \Rightarrow \sigma_{Cij}^2 = \sum_{i=1}^{nT} (n_{bi} + n_i) / \alpha \beta (1-\beta) + n_{px} \sigma_D^2 \cdot (\beta^2 + (1-\beta)^2) / \alpha^2 (1-\beta)^2 \beta^2$$

The fluxes of the two beams are approximately equivalent: $n_{bi}=n_{bj}=nb$ and $n_i=n_j=n$

$$\sigma_{Cij}^2 = n_T(n_b+n) / \alpha \beta (1-\beta) + n_{px} \sigma_D^2 \cdot (\beta^2 + (1-\beta)^2) / \alpha^2 (1-\beta)^2 \beta^2$$

In the case of thermal background regime:

$$\sigma_{Cij}^2 = n_T \cdot n_b / \alpha \beta (1-\beta) \quad (7)$$

- “Crosstalk between peak” case: It case corresponds to a fringe peak with contamination by the thermal background and the energy peaks : $M_b(u_{ij}) \neq 0$ and $M(u_{ij}) \neq 0$.

$$E(C_{ij}) = I(u_{ij}) / \alpha \beta - \sum_{i=1}^{nT} \frac{P_i(u_{ij})}{(1-\alpha)\beta}$$

Leading to a variance σ_{Cij}^2 ,

$$\sigma_{Cij}^2 = \sigma_I^2 / \alpha^2 \beta^2 + \sum_{i=1}^{nT} \frac{\sigma_{Pi}^2}{(1-\alpha)^2 \beta^2} \Rightarrow \sigma_{Cij}^2 = \sum_{i=1}^{nT} (n_{bi} + n_i) / \alpha \beta (1-\alpha) + n_{px} \sigma_D^2 \cdot (n_T \cdot \alpha^2 + (1-\alpha)^2) / \alpha^2 (1-\alpha)^2 \beta^2$$

The fluxes of the two beams are approximately equivalent: $n_{bi}=n_{bj}=nb$ and $n_i=n_j=n$

$$\sigma_{Cij}^2 = n_T(n_b+n) / \alpha \beta (1-\alpha) + n_{px} \sigma_D^2 \cdot (n_T \cdot \alpha^2 + (1-\alpha)^2) / \alpha^2 (1-\alpha)^2 \beta^2$$

In the case of thermal background regime:

$$\sigma_{Cij}^2 = n_T \cdot n_b / \alpha \beta (1-\alpha) \quad (8)$$

3.3 Photometric flux estimation

The photometric information is extracted from the photometric channels by the following estimator :

$$E(n_i) = P_i(0)/(1-\alpha)\beta - P_{i,sky}(0)/(1-\alpha)(1-\beta)$$

The variance of this estimator is :

$$\sigma_{ni}^2 = \sigma_{Pi}^2/(1-\alpha)^2\beta^2 + \sigma_{Pi,sky}^2/(1-\alpha)^2(1-\beta)^2$$

With σ_p^2 (variance of each photometric image) :

$$\begin{aligned} \sigma_{Pi}^2 &= (1-\alpha)\beta(n_{bi}+n_i) + n_{px}\sigma_D^2 & \sigma_{Pi,sky}^2 &= (1-\alpha)(1-\beta)(n_{bi}) + n_{px}\sigma_D^2 \\ & & \Rightarrow & \\ \sigma_{ni}^2 &= n/(1-\alpha)\beta + n_b/(1-\alpha)\beta(1-\beta) + n_{px}\sigma_D^2/\beta \end{aligned}$$

In the case of thermal background regime:

$$\sigma_{ni}^2 = n_b/(1-\alpha)\beta(1-\beta) \quad (9)$$

4. SNR COMPARISON BETWEEN THE PAIRWISE 0- π AND THE GLOBAL CONCEPTS

The visibility estimator is:

$$E(V_{ij}) = E(C_{ij})/(E(n_i)E(n_j))^{1/2}$$

Its root mean square is:

$$\sigma_{Vij} = \sigma_{Cij}/(n_i n_j)^{1/2} + \sigma_{ni} C_{ij}/(n_i n_j)$$

It requires the knowledge of σ_{Cij}^2 and σ_{ni}^2 .

The equations (7) and (8) show that the optimal SNR on the coherent flux for the global case (if the fringe peak is contaminated) is obtained with $\alpha=0.5$ (equivalent flux for the photometric and the interferometric channel) or $\beta=0.5$ (equivalent time spent on the object and on the sky).

The following comparison is made with these values, except for the pair-wise 0- π combination case with 3 or 4 telescopes where the photometry can be deduced from the interferometric signal ($\alpha=1$).

The following tables give the inverse of σ^2 (which represent the square of the signal to noise ratio) of the coherent and photometric fluxes relative to the 4 telescopes global case taken as a reference.

Coherent flux table

$1/\sigma_{Cij}^2$	2T	3T	4T
Global "Thermal background contamination peak": $\alpha=0.5, \beta=0.5$	2	4/3	1
Global "Crosstalk between peak": $\alpha=0.5, \beta=0.5$	2	4/3	1
Global "Clear peak": $\alpha=0.5, \beta=0.5$	4	8/3	2
Pair wise 0- π : $\alpha=0.5, \beta=0.5$	4	-	-
Pair wise 0- π : $\alpha=1, \beta=0.5$	-	4	8/3

Photometry table

$1/\sigma_{ni}^2$	2T	3T	4T
Global "Thermal background contamination peak": $\alpha=0.5, \beta=0.5$	1	1	1
Global "Crosstalk between peak": $\alpha=0.5, \beta=0.5$	1	1	1
Global "Clear peak": $\alpha=0.5, \beta=0.5$	1	1	1
Pair wise $0-\pi$: $\alpha=0.5, \beta=0.5$	1	-	-
Pair wise $0-\pi$: $\alpha=1, \beta=0.5$	-	2/3	8/15

5. CONCLUSION

The pairwise $0-\pi$ concept without specific photometric channel offers the best solution for different reasons:

- A better sensitivity (SNR on coherent flux),
- A better reliability for extracting fringes from background, benefits from MIDI experience using the $0-\pi$ mode,
- A better visibility accuracy (from the SNRs achieved on the photometry and the coherent flux) ,
- A reasonable optical complexity.

A pair-wise separation and a $0-\pi$ splitting can be performed with a multi-axial combination of the 2 beams contained in each pair. This kind of combination automatically transforms the temporal scanning into spatial encoding and allows a larger margin of the detector readout time. It is thus more compatible with the detector readout speed considering the coherence time of the atmosphere. It offers the advantage to avoid temporal scanning mandatory in the co-axial mode and the related difficulty to implement the actuators after the pair-wise separation in the MATISSE cryostat. The possibility to perform temporal modulation in the warm optics is however maintained to allow a better data calibration by regards to background fluctuations and detector gain table variations.

REFERENCES

- [1] Lagarde, S & Lopez, B., "MATISSE system design", ESO document VLT-TRE-MAT-15860-0004, 2007.
- [2] Robbe-Dubois, S., Lagarde S. et al, "AMBER: optical analysis and configuration", A&A, Vol 464, pp. 13-27, 2007.
- [3] Ch Leinert, U Graser et al., "MIDI - the 10 μm instrument on the VLTI", Astrophysics and Space Science, pp. 73-83, 286 (1-2), 2003.